Towards Robust Inference in Graphical Models

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Beforehand

- This is ongoing work
  - Speculative statements: *
  - Unsubstantiated statements: *
- Informal development
- Toy problems
- Feedback and collaboration welcome

Outline

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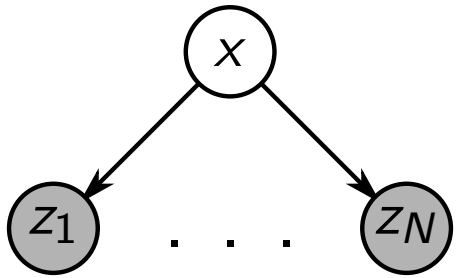
Inference in Graphical Models

- 2 components: model and evidence
- Modelling:
  - Define random variables and statistical relationships between them
  - Directed graph (Bayes network) or indirected graph (Markov network)
  - General framework
- Evidence
  - Typically measurements
- Inference: model + evidence $\rightarrow$ answers to questions
  - 2 main flavours: variational and sampling-based
  - We focus on message passing (variational)

Regression Example

Model:

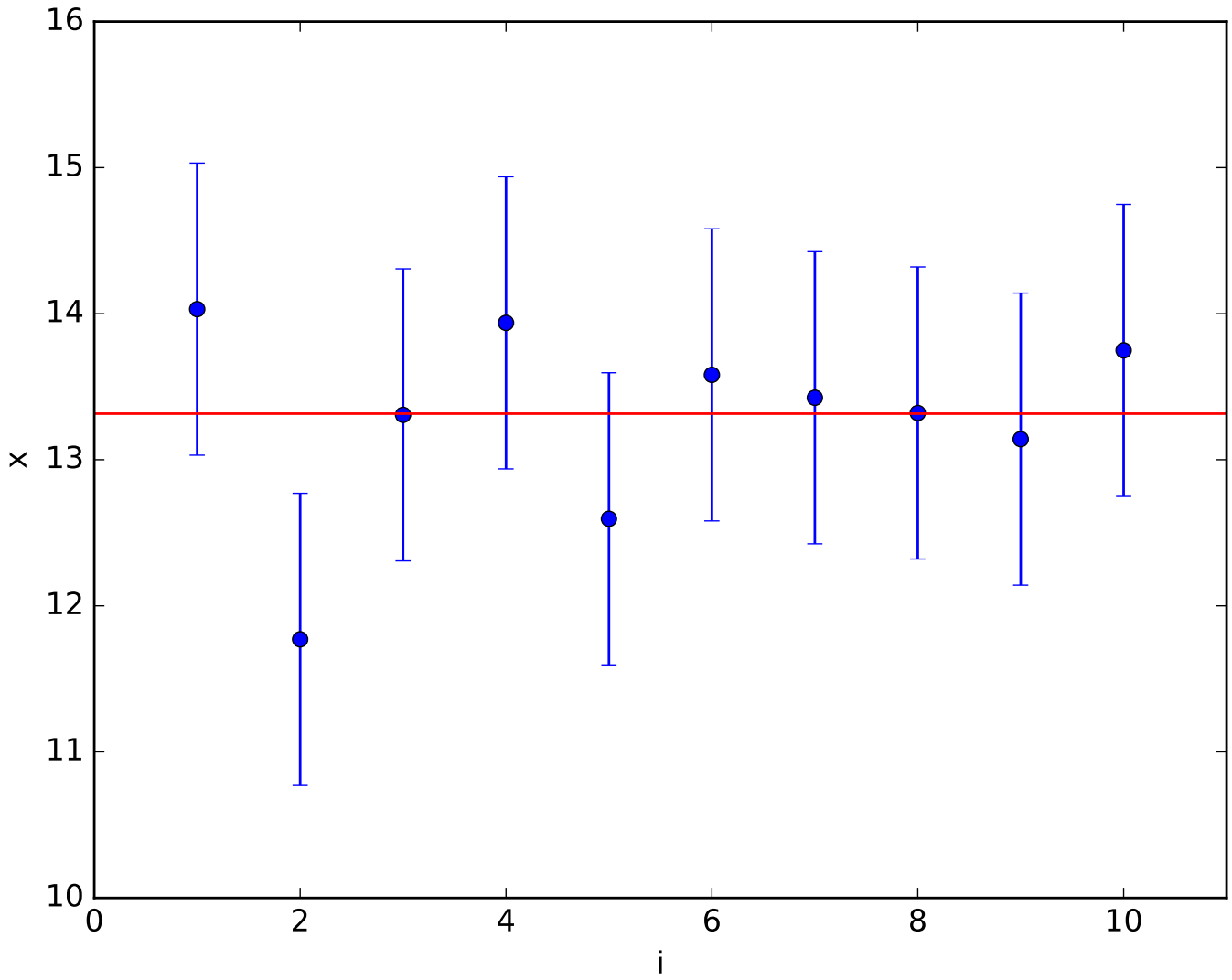
$$p(x) = \mathcal{N}\left(\mu_x, \sigma_x^2\right)$$
$$z_i = x + \epsilon_i, \quad \epsilon_i \sim \mathcal{N}\left(0, \sigma_z^2\right), \quad i = 1, \ldots, N$$



Question:

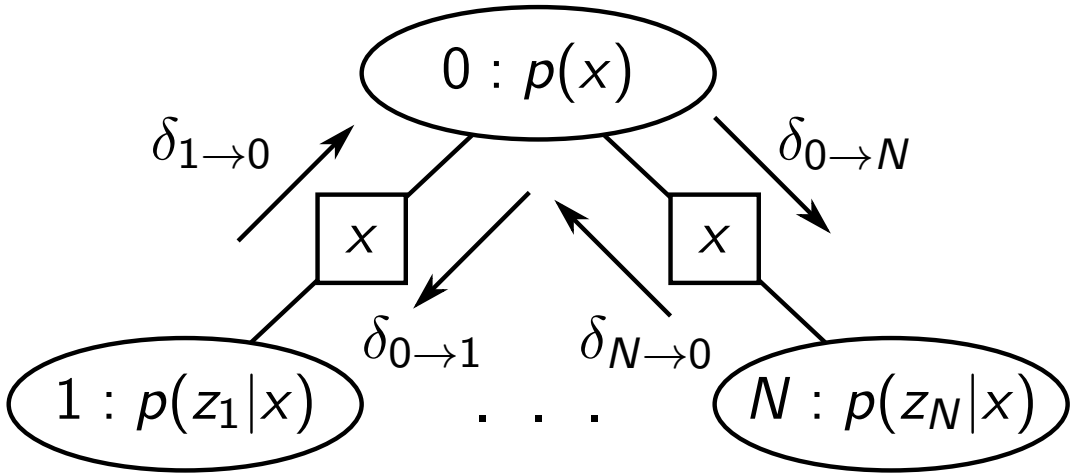
$$p\left(x | z_1, \ldots, z_N\right)?$$

Regression Example



Regression Example

Inference:



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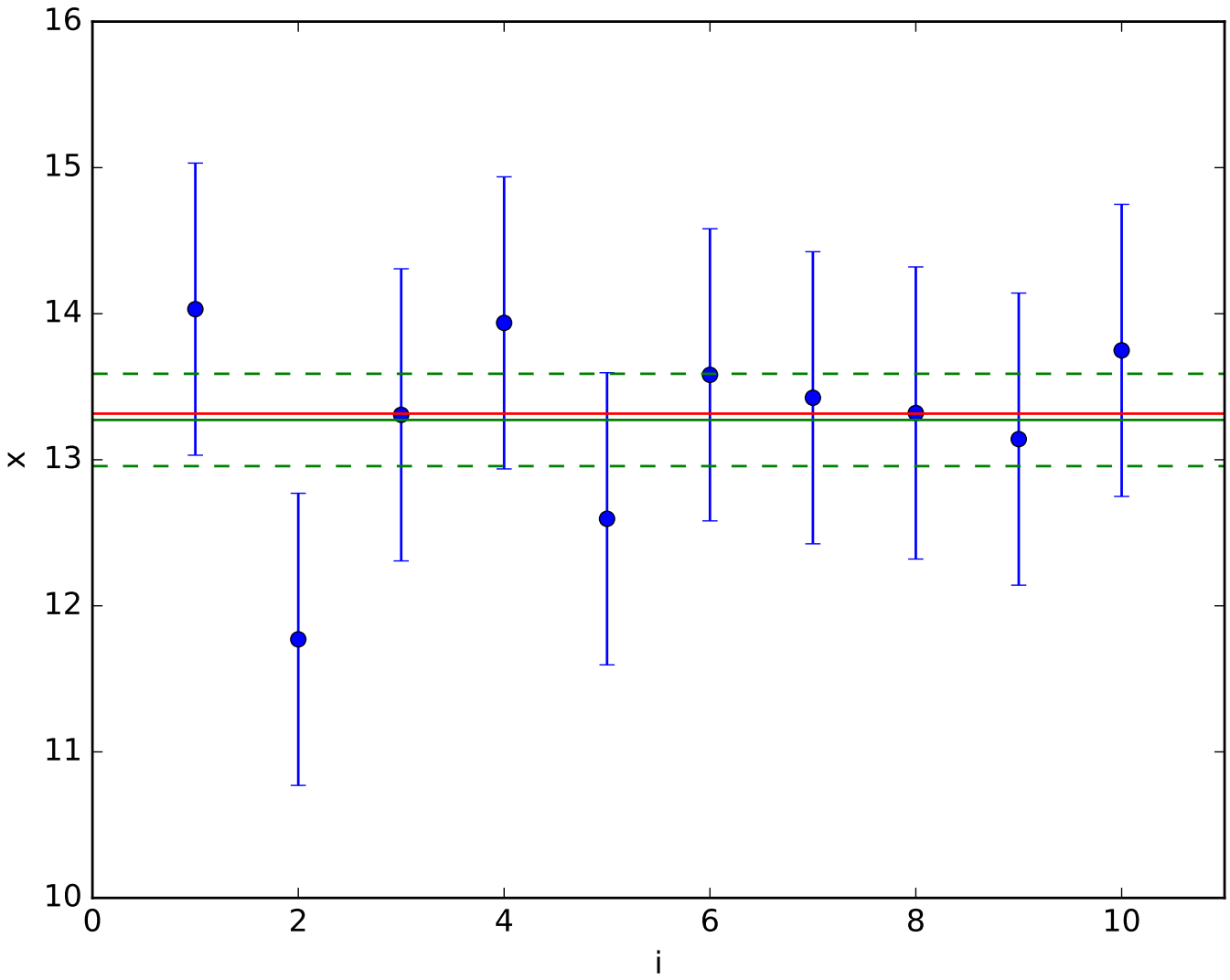
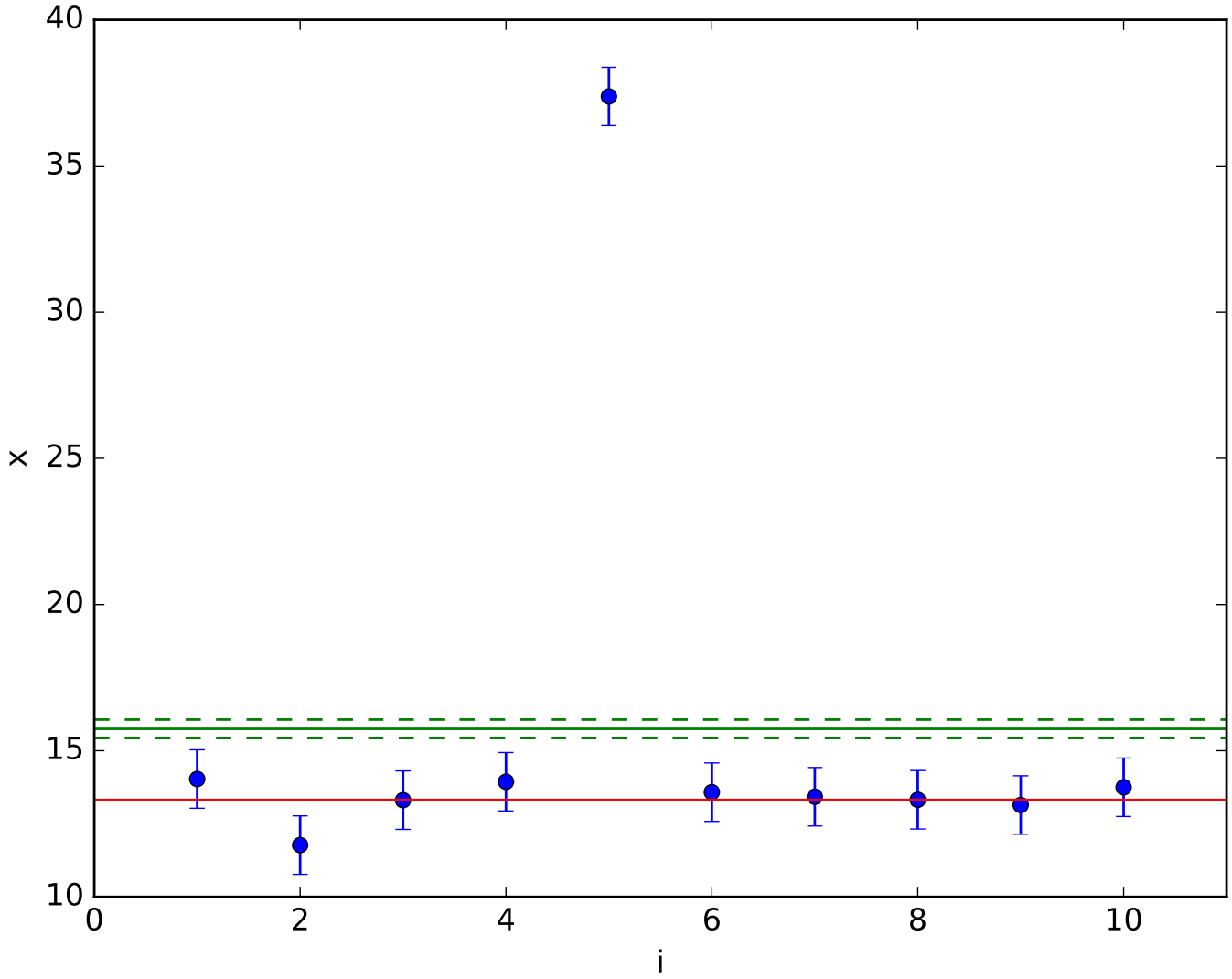


Outline

So what is the problem?

- Inference never produces correct answers
  - Models are never correct*
  - Tractable inference require approximations
- Result:
  - Rubbish answers
  - Things break*
- Existing approaches:
  - Full Bayesian approach
  - Model selection
- Disadvantages of existing approaches:
  - Require complex models
  - Expensive
  - They do not solve problem*

Regression Example



Outline

Quantifying Model Inconsistency

- Bayesian surprise:

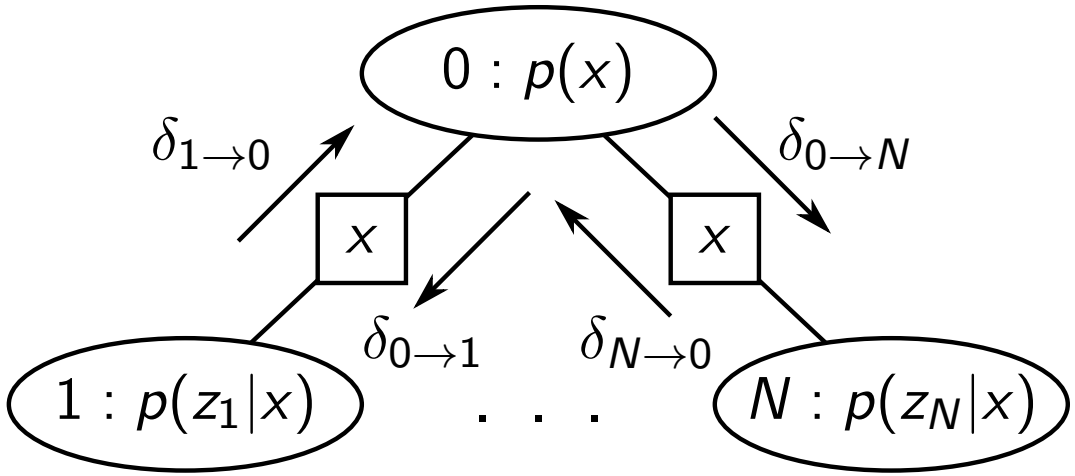
$$S(E, p(x)) \triangleq D_{KL}(p(x|E)||p(x))$$

  - *How much the evidence changed the prior belief*
- Properties:
  - Number between 0 and ∞
  - Lower measurement likelihood $\rightarrow$ higher surprise
  - Defined for most parametric distributions
  - For Gaussian measurement – scaled Mahalanobis distance squared

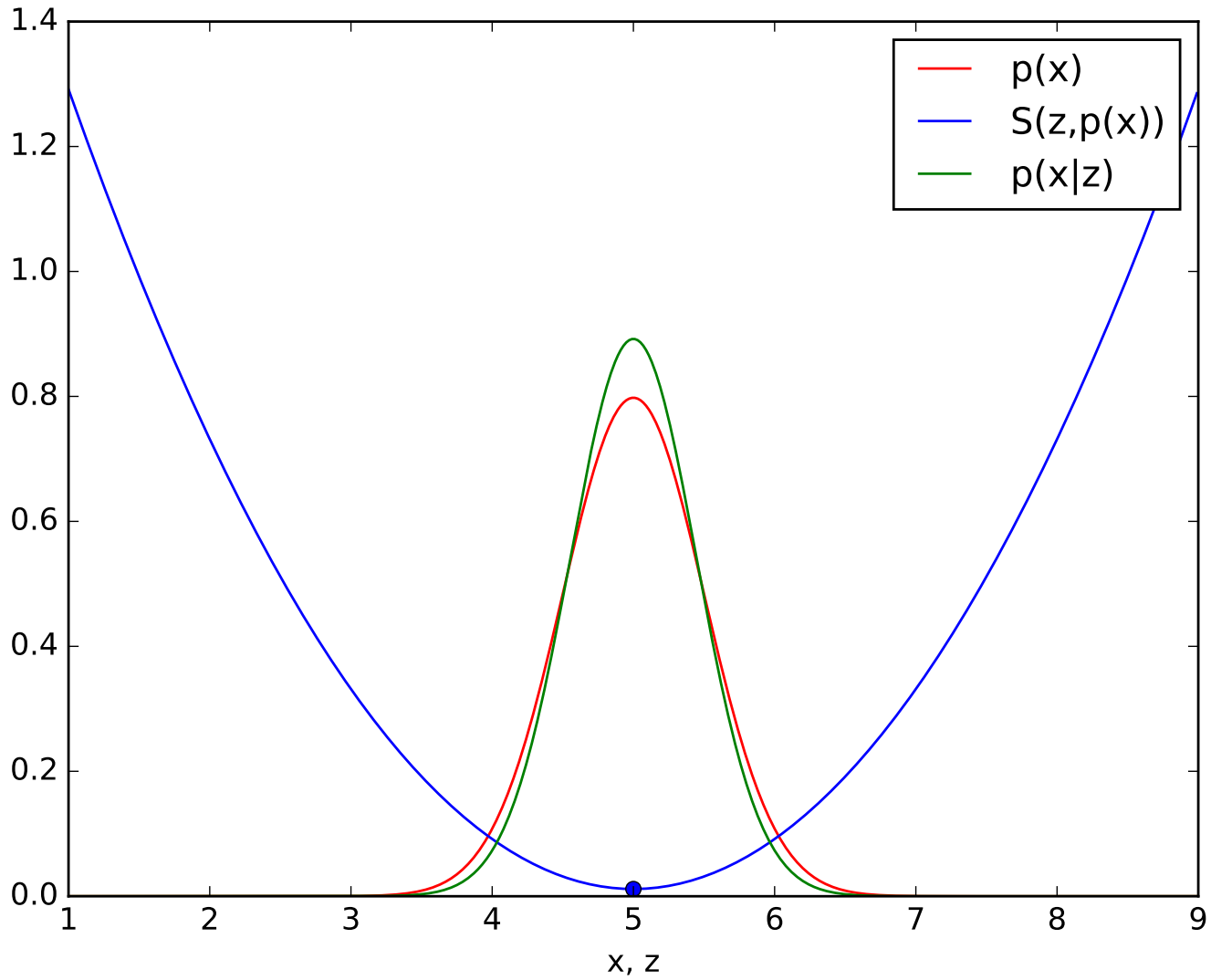
Quantifying Model Inconsistency

- For regression example:

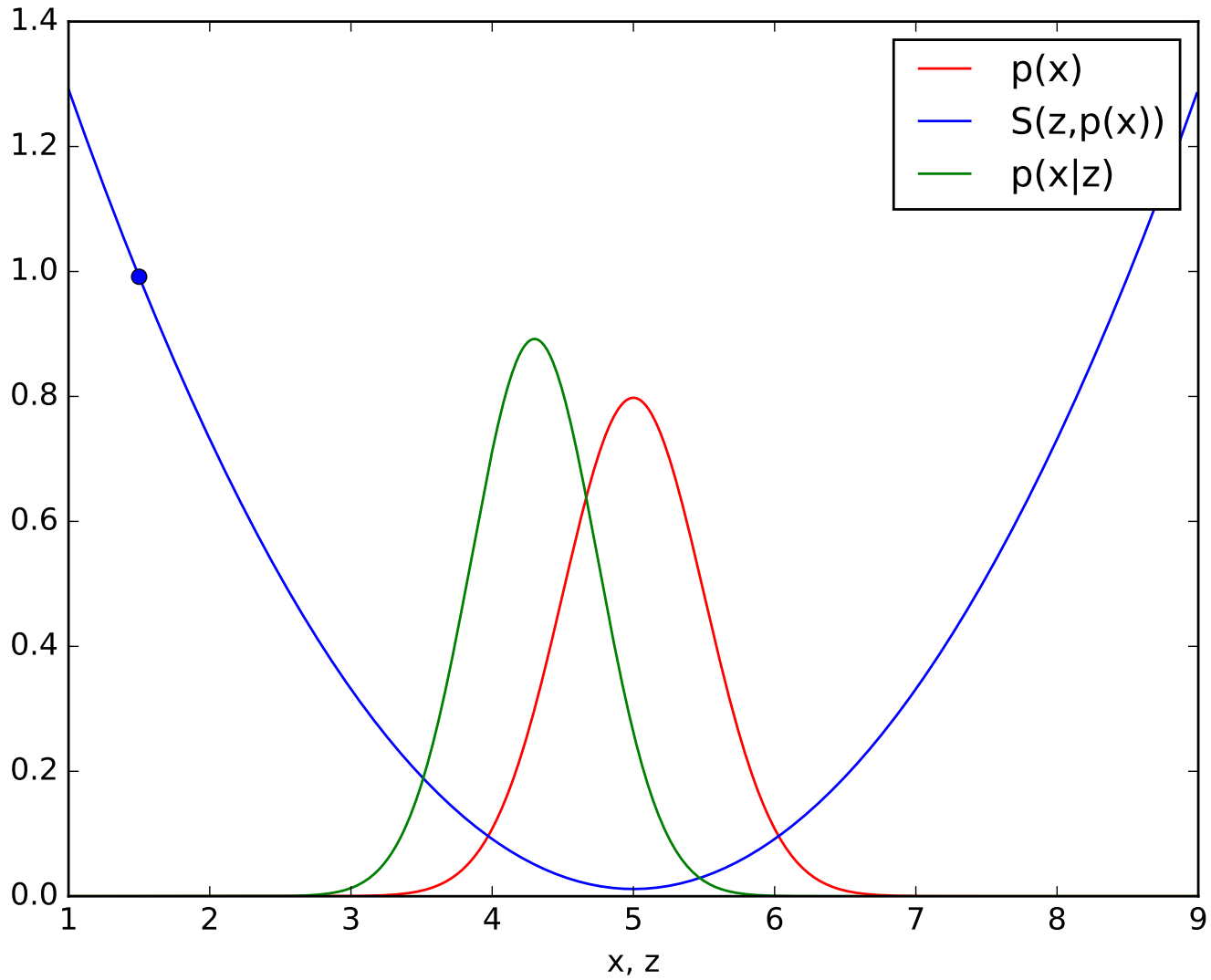
$$S_1 \triangleq S\left(z_1, \delta_{0\to i}(x)\right) = D_{KL}\left(\beta(x)||\delta_{0\to 1}(x)\right)$$



Quantifying Model Inconsistency



Quantifying Model Inconsistency



Quantifying Model Inconsistency

- Conflict between whole model and evidence:

$$C = \sum_i S_i$$

- What level of conflict is acceptable?
  - Calculate $\mathcal{E}[C]$ and $var[C]$ using model
  - Assume Gaussian distribution
  - Calculate threshold C_{th} such that $P[C \leq C_{th}] = x\%$

Reducing Model Inconsistency

- Factor relaxation
  - Introduce adjustable parameters $\boldsymbol{\theta}_i$
  - If $\boldsymbol{\theta}_i = \mathbf{0}$, factor unchanged
  - As $\boldsymbol{\theta}_i$ increase, surprise decreases
- Relaxation for regression example:

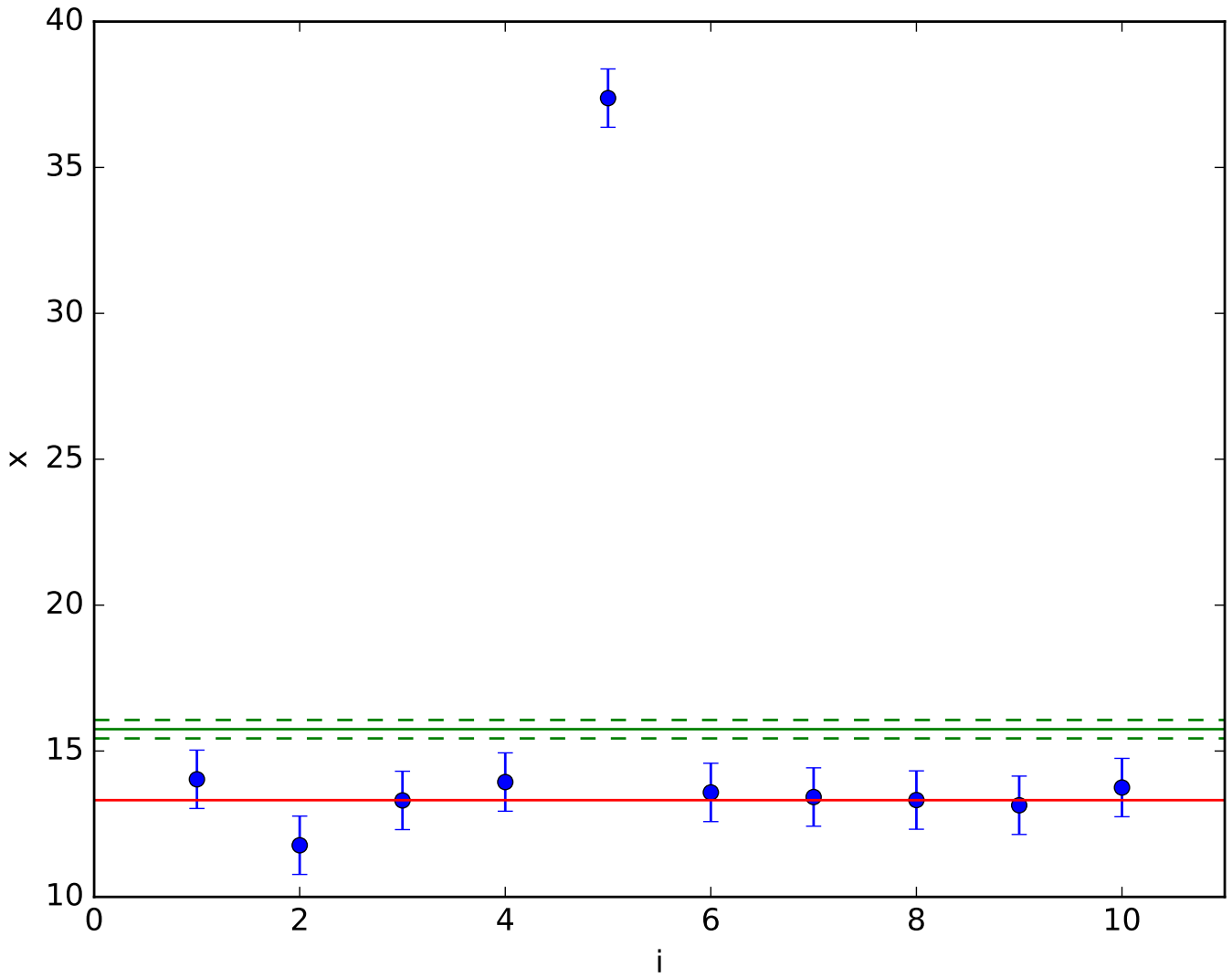
$$z_i = x + \epsilon_i, \quad \epsilon_i \sim \mathcal{N}\left(0, (\sigma_z + \theta_i)^2\right)$$

- Relaxation cost: $J(\boldsymbol{\theta})$
  - E.g. $J(\boldsymbol{\theta}) = \|\boldsymbol{\theta}\|_1$

Reducing Model Inconsistency

- Problem formulation:
  - Minimise $J(\boldsymbol{\theta})$ s.t. $C \leq C_{th}$
  - Or: *find the cheapest consistent model*
- Minimisation process:
  - Calculate $\frac{\partial C}{\partial \boldsymbol{\theta}}$ using automatic differentiation
  - Use modified Newton's method

Regression Example



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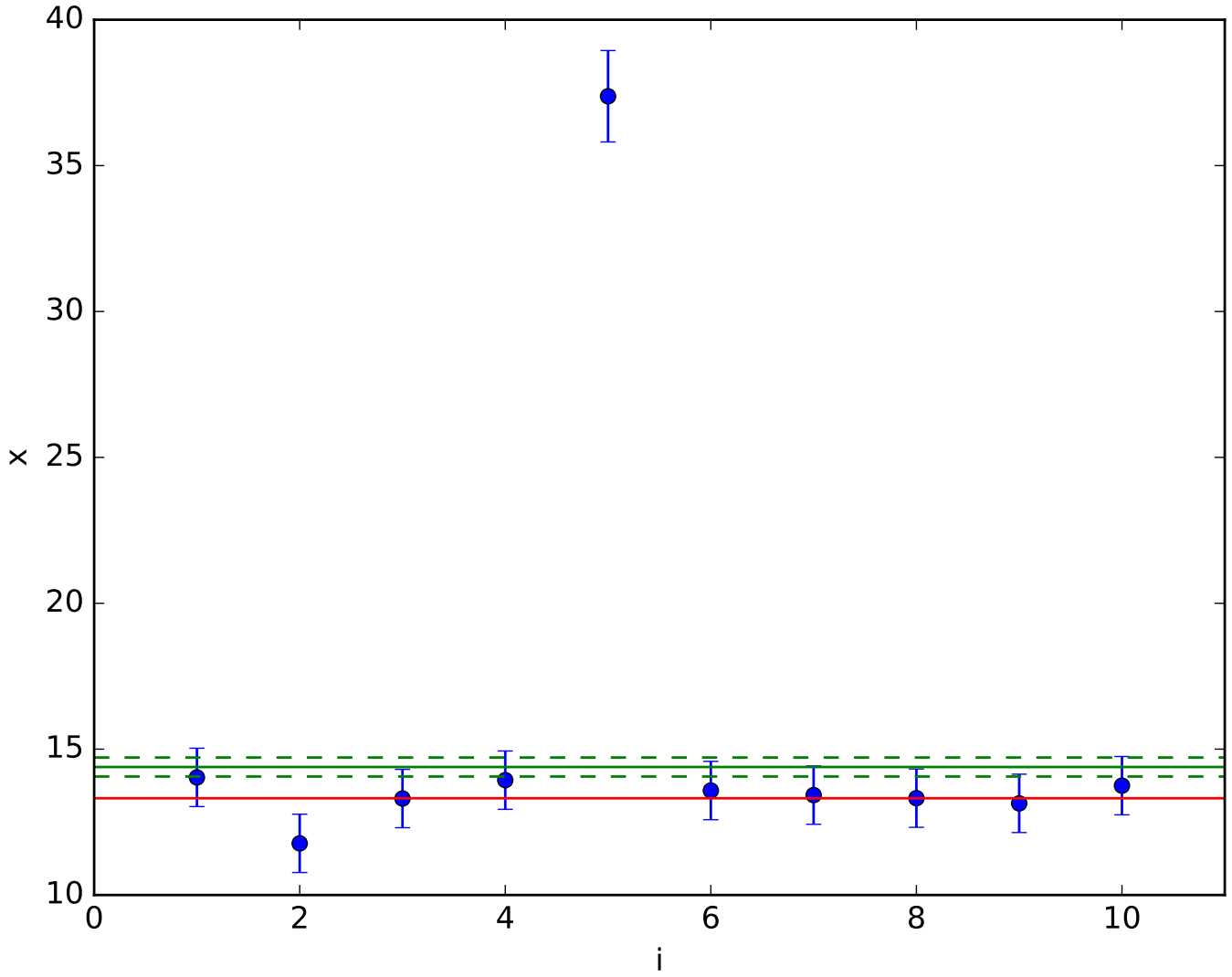

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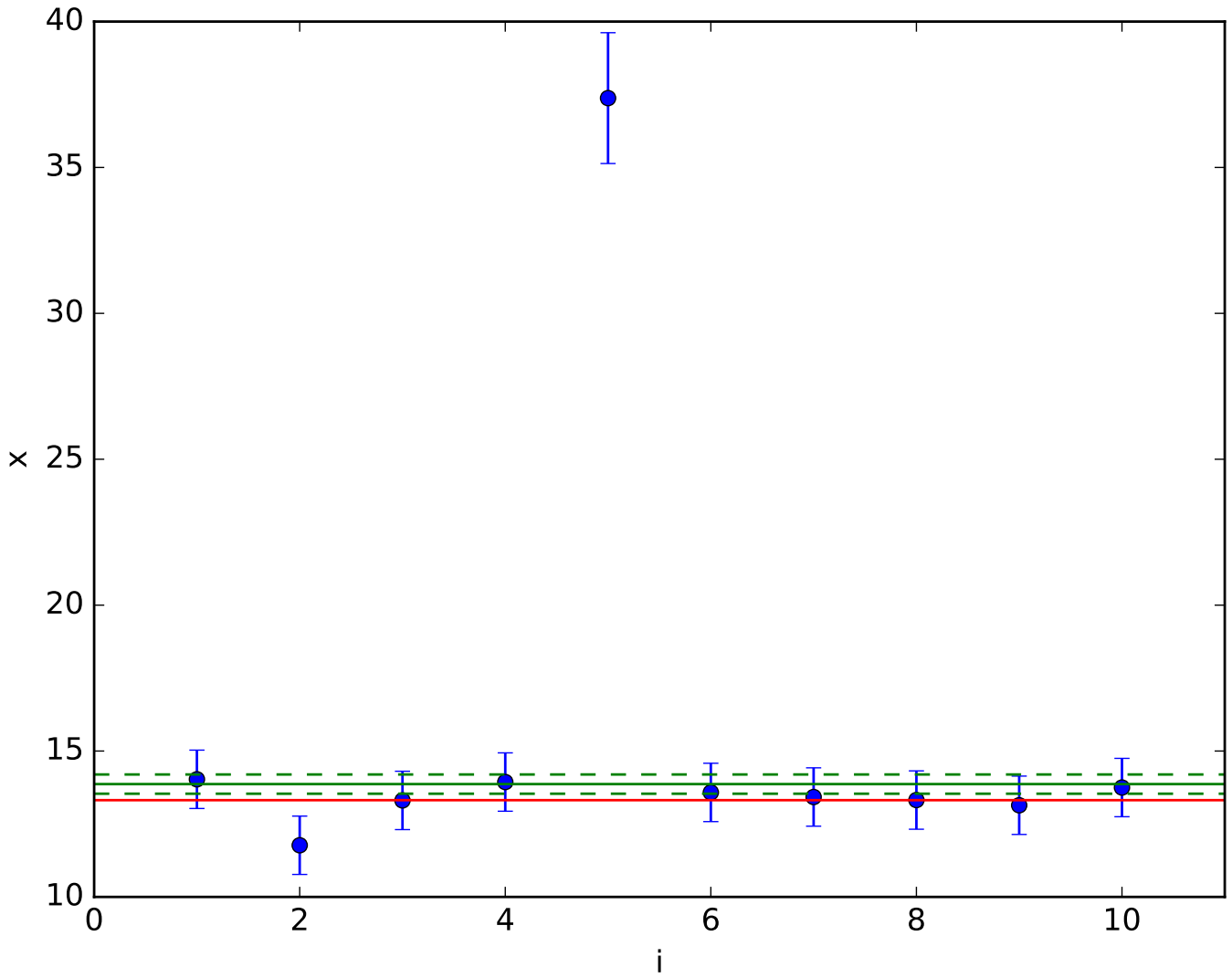

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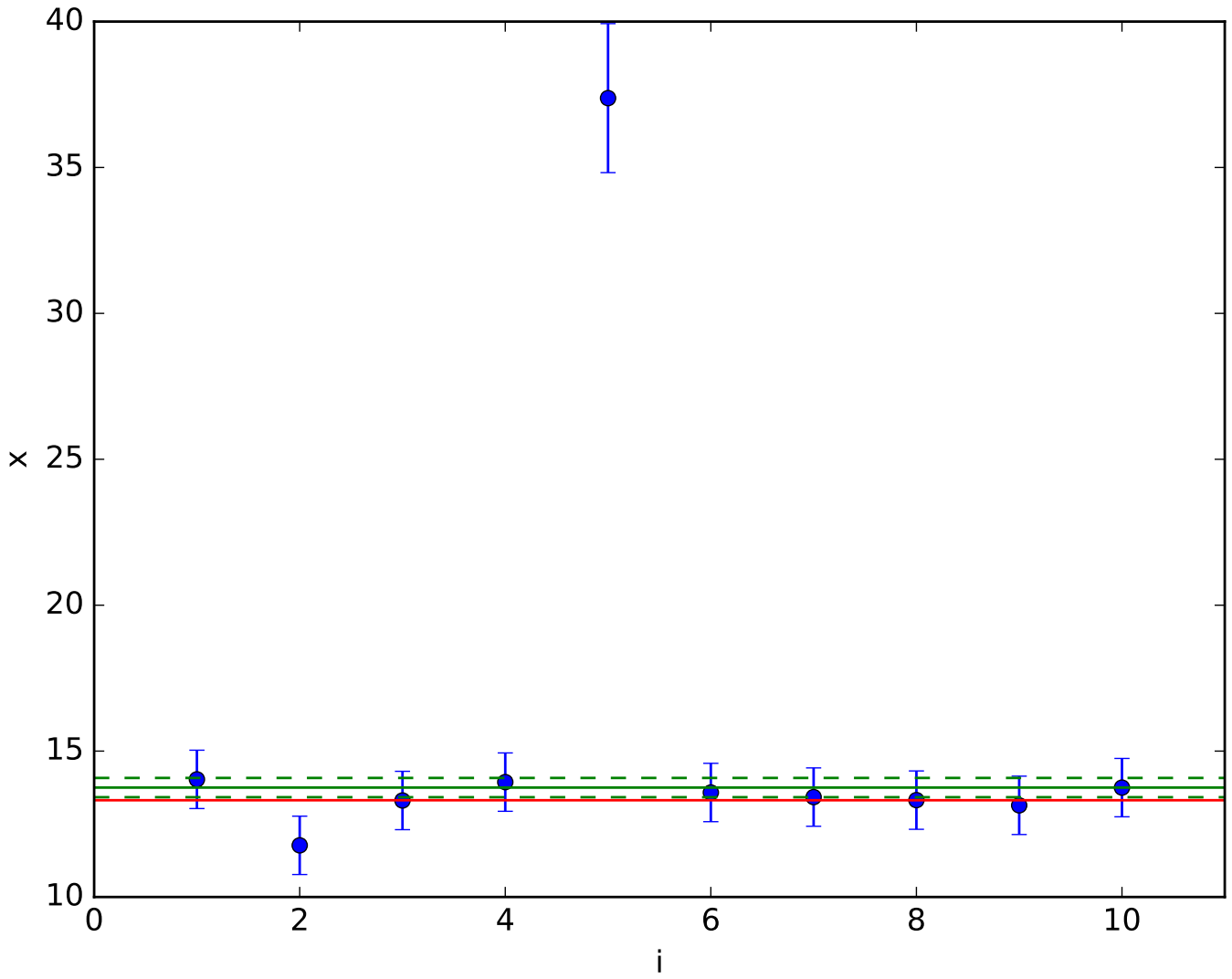
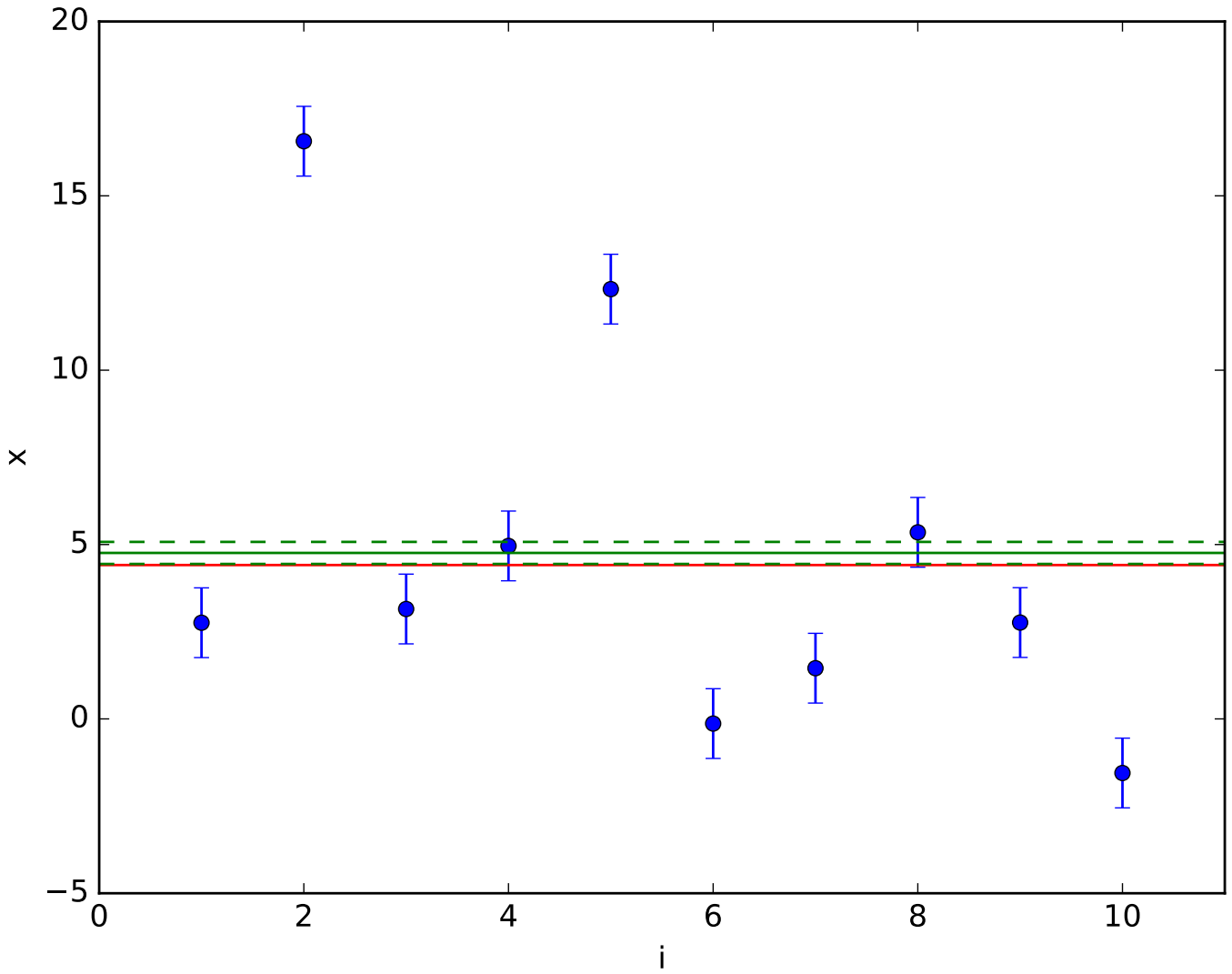


Regression Example

But why do we not just use outlier detection and removal?

Regression Example



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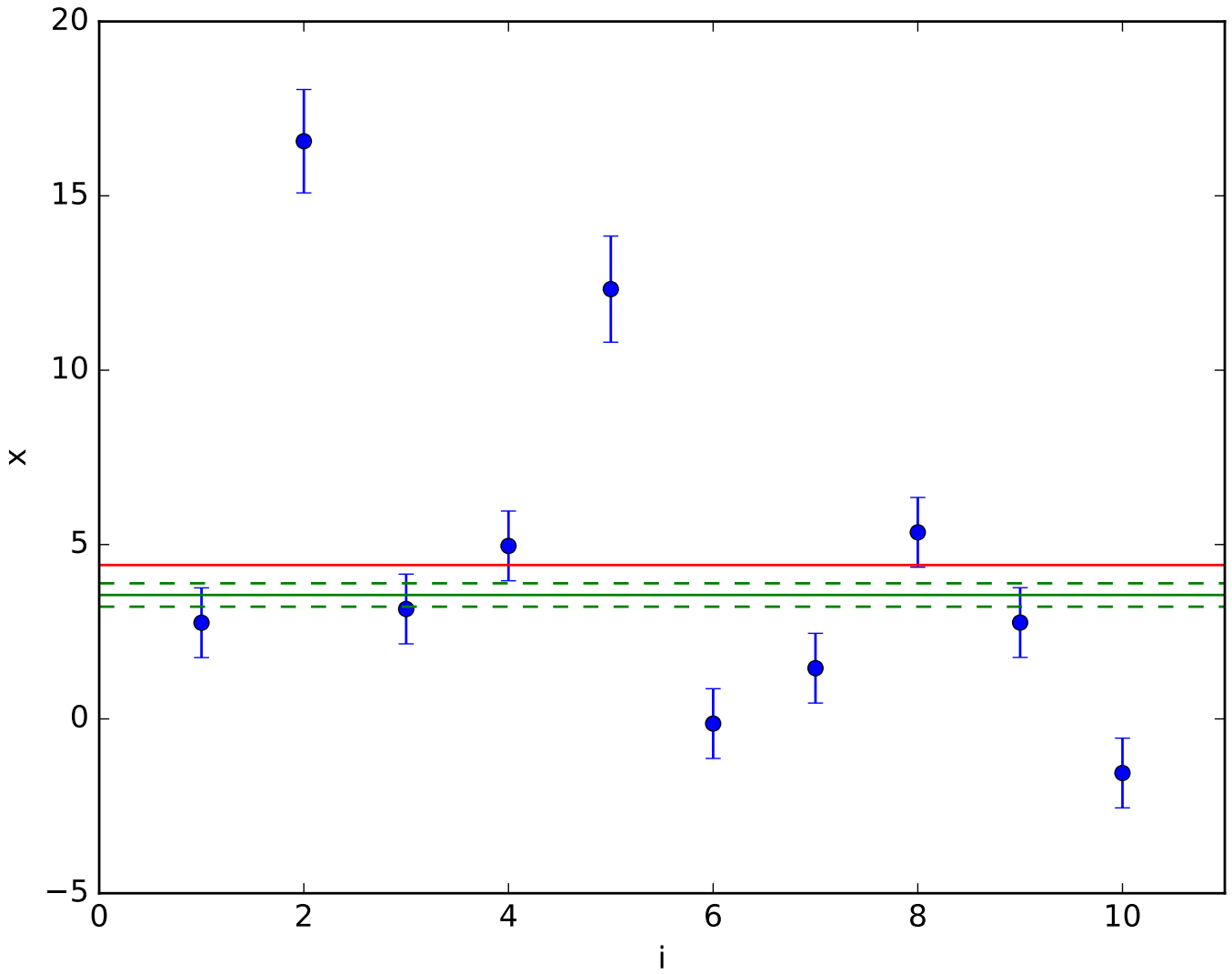

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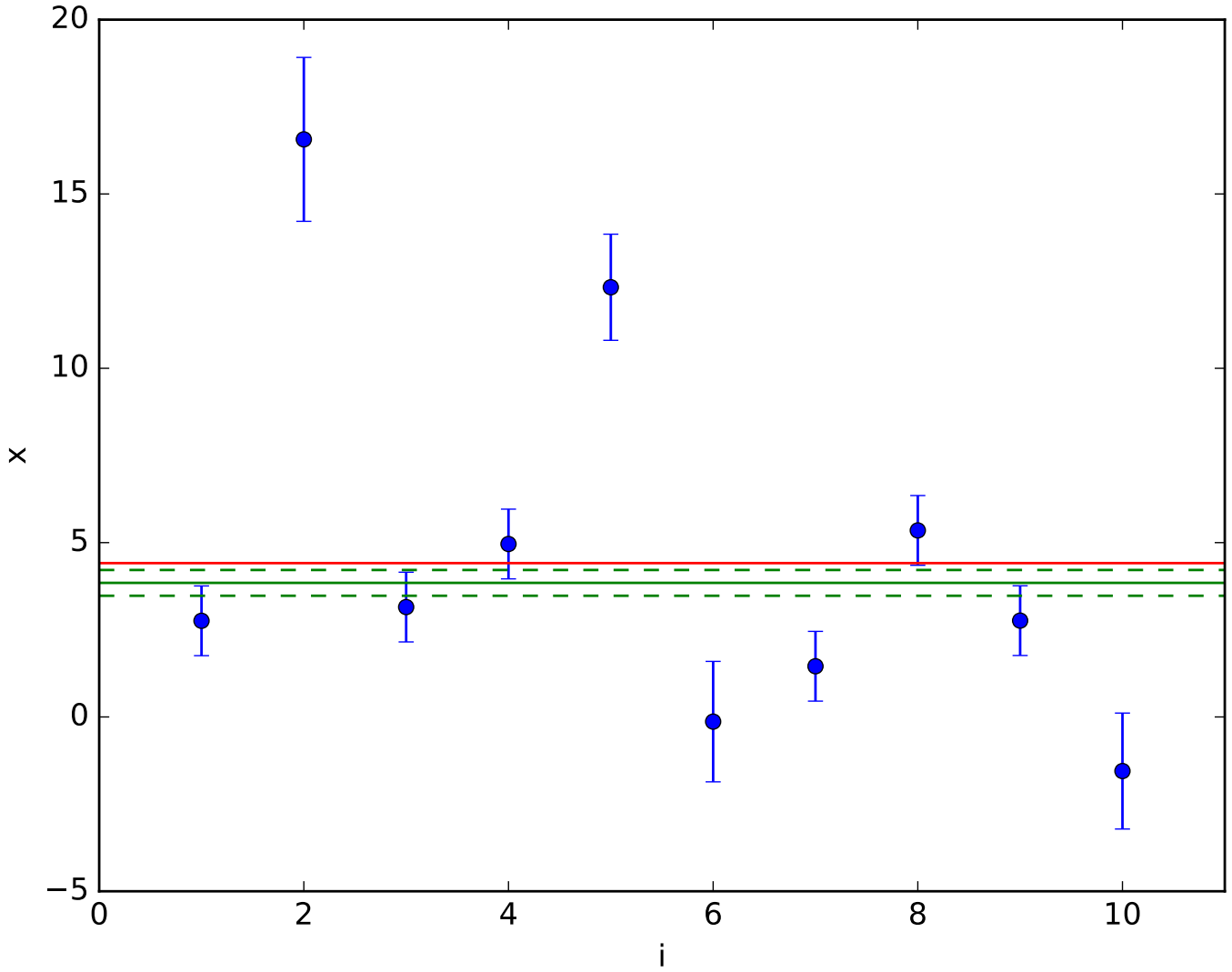

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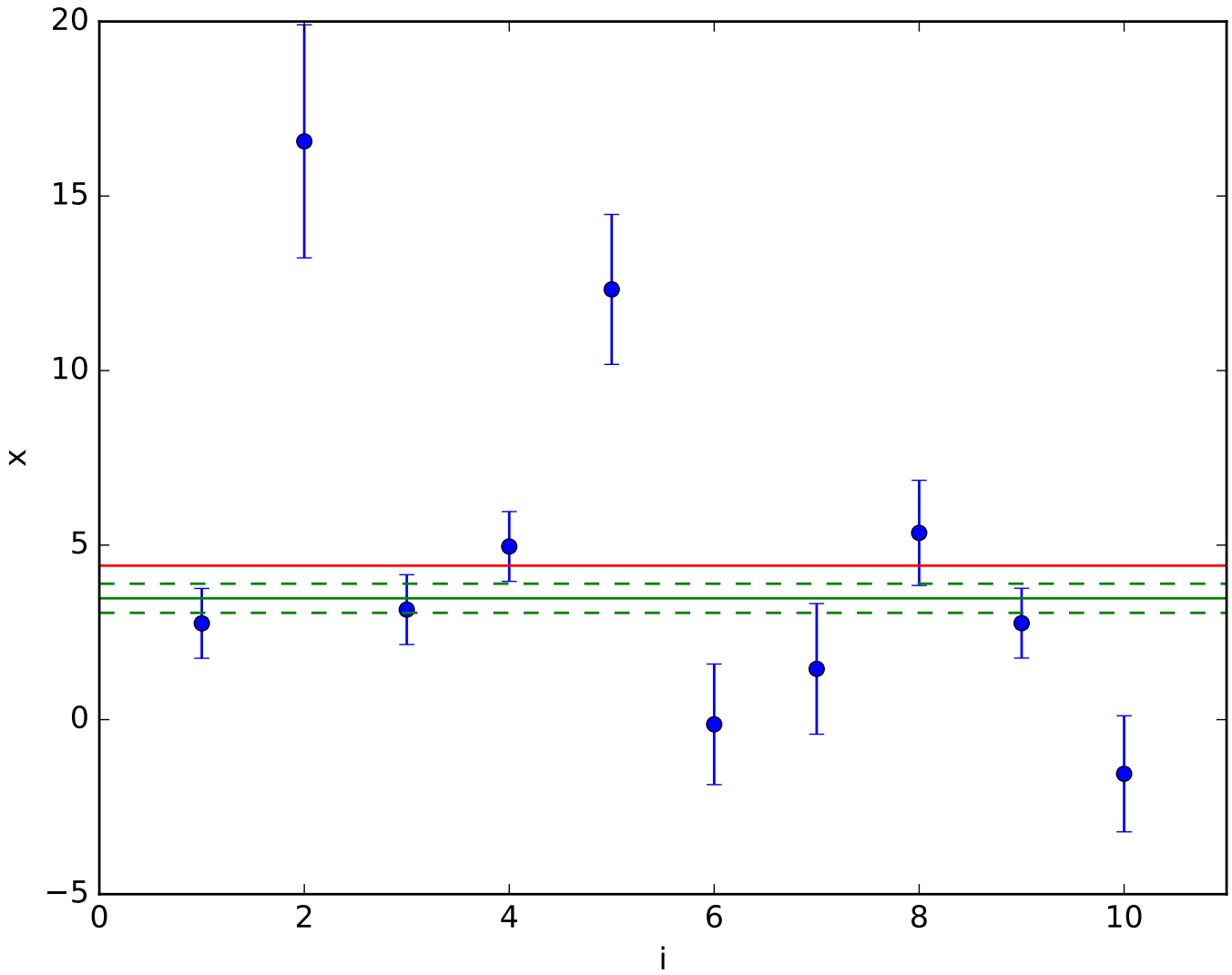

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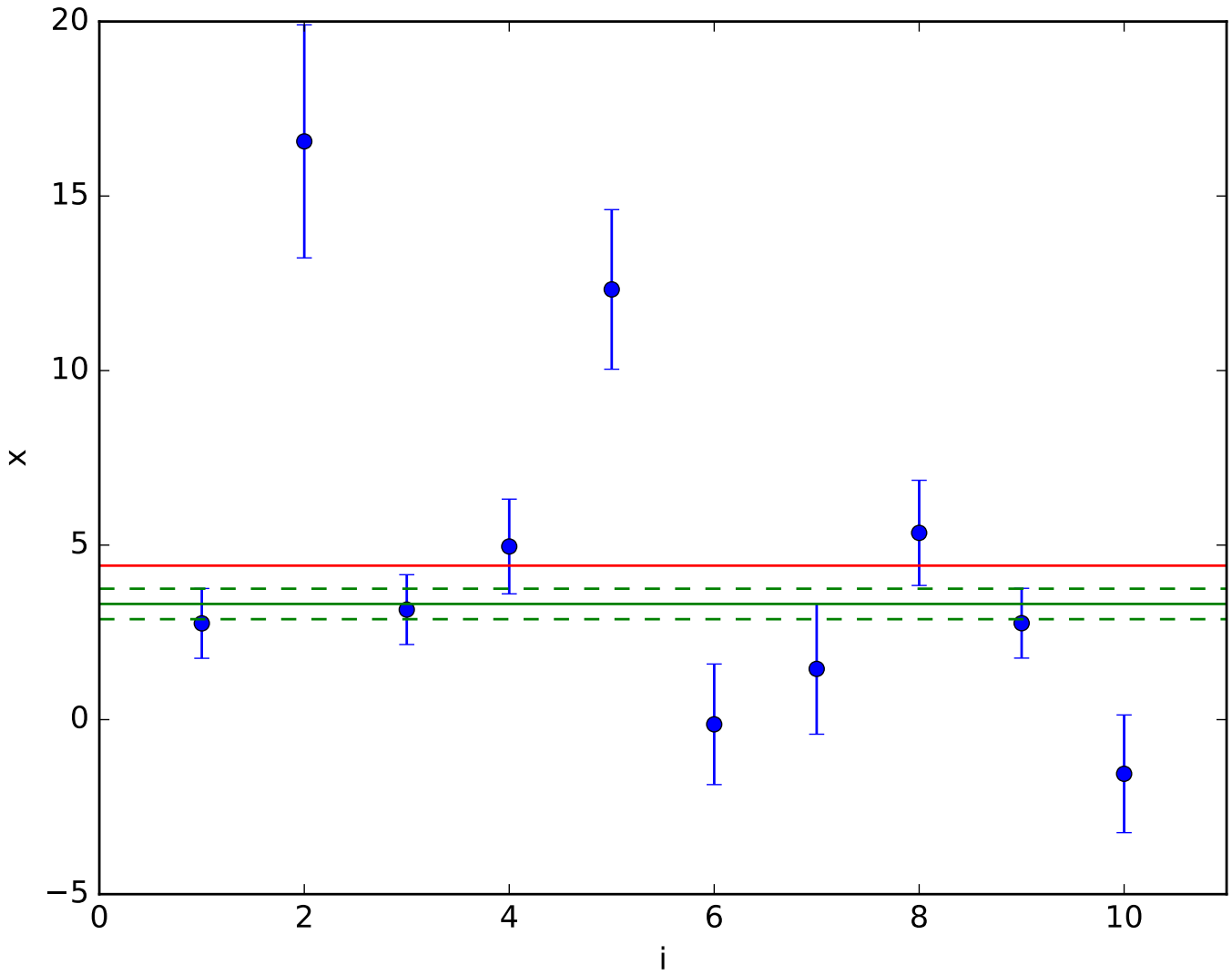

Outline

Conclusion

- Summary:
  - Inconsistency between model and evidence cause problems
  - We propose a method to relax a model during inference to reduce model inconsistency
  - Which does not rely on complex, accurate modelling
  - This produces better wrong answers*
- Characteristics of proposed method:
  - Fits into message passing for graphical models
  - Applicable to common continuous and discrete factors*
  - Can be extended to general factors*
  - Applicable to loopy networks*
- Further ideas and work:
  - Test for various, realistic problems
  - Extend to loopy networks
  - Default reasoning